For various flow classes, this rank can be 1, 2, or 3. Even if the rank is 3, the problem can be very close to a singular one. $Q_w(X)$ nonlinearly depends on ($\rho_\infty U_\infty T_\infty$) and nonlinearly varies along $X$. The qualitative analysis shows that it would be appropriate to measure heat flux at points with maximum different flow type. The corresponding quantitative criteria are the determinant and the conditionality of the sensitivity matrix $A_{ij}$ and the spectrum of eigenvalues $\lambda_i$ of Fisher's informational matrix ($M_{jm} = A_{ij} A_{im}$). The numerical experiments and experimental data show that the problem condition can be improved by the location of the measurements in zones of different flow type (at the plate and the edge, for example). Therefore, the measurement points can be chosen before experiment by the optimization of the determinant or the conditionality.

The results of the Ref. 3 experimental data treatment confirm the feasibility of $\rho_\infty U_\infty$ and $T_\infty$ determination using laminar heat fluxes. The estimation of two parameters for a flat plate and three parameters for a corner is feasible.

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# Limiting Mach Number for Quantitative Pressure-Sensitive Paint Measurements

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## Introduction

THE pressure-sensitive paint (PSP) technique for measuring surface pressure and obtaining coefficient of pressure ($C_P$) data has recently gained acceptance as an alternative and complement to traditional instrumentation systems and computational techniques. The PSP technique is based on the pressure sensitivity of photoluminescent materials and in its simplest form may be represented by an equation relating a reference wind-off to wind-on photoluminescence intensity

$$I_0/I = A + B(P/P_0) \qquad (1)$$

where $I$ is the PSP's photoluminescent signal, $P$ is the surface pressure, $A$ and $B$ are the PSP Stern–Volmer sensitivity coefficients, and the subscript refers to a reference level. The fundamentals and several applications of PSP measurements based on Eq. (1) are introduced by Morris et al.[1] and reviewed by McLachlan and Bell.[2] A detailed development of the PSP governing equations, including Eq. (1), and a comprehensive uncertainty analysis of the PSP instrumentation system are presented in Mendoza.[3] This last study calculated uncertainty ($dC_P$) of PSP derived $C_P$ data by dividing PSP instrumentation system error sources into three categories including the PSP methodology, PSP implementation, and PSP imaging system. The random and bias error components corresponding to these categories were obtained by using representative noise values characteristic of first generation (typical) and state-of-the-art PSP instrumentation systems. [Typical systems are those where PSP photoluminescence is not corrected for significant influencing parameters other than pressure (e.g., temperature, photodecomposition, and excitation fluctuations) with a nonscientific grade imager, and where geometrical and optical inconsistencies between wind-on and wind-off images used in Eq. (1) are not corrected for.[3] With state-of-the-art systems, PSP photoluminescence is not significantly influenced by parameters other than pressure or is corrected for them, with a scientific grade imager (e.g., 14-bit A/D, low read out noise and low dark current, high capacity quantum wells), and the geometrical and optical inconsistencies between wind-on and wind-off images used in Eq. (1) corrected for.[3]] These errors were propagated into $C_P$ calculations using a linearized Taylor series, as outlined in Bevington and Robinson,[4] with the additional assumption that error sources were independent such that all cross correlation terms in the series were zero. Final $dC_P$ values were calculated using the rss uncertainty model corresponding to a 95% coverage level when the component bias and random errors are added by the rms method. These $dC_P$ results were presented as a function of freestream Mach number ($M_\infty$) and $C_P$. Additionally, these results identified critical sources of noise that must be addressed to minimize PSP measurement uncertainty.

Mendoza's analysis reemphasized the conclusion presented by Sajben[5] that PSP instrumentation systems were better suited for moderate to high Mach number flows. Mendoza showed that the primary reason for this was that $dC_P$ scales approximately as one over the Mach number squared, manifesting into large errors in calculated $C_P$ at low Mach numbers. Sajben's analysis was constrained to ideal test conditions such as uniform model surface temperature and the assumption that photodegradation of the PSP coating does not occur; Mendoza's was not. However, neither analysis determined the limiting $M_\infty$ for obtaining quantitative PSP measurements with respect to traditional instrumentation systems (force balances and pressure taps).

This analysis develops a technique to determine the limiting $M_\infty$ for acquiring quantitative PSP measurements of pressure. The analysis is based on defining $C_P$ uncertainties in terms of coefficient of lift ($c_l$) uncertainties ($dc_l$). The $dC_P$ for quantitative PSP measurements will then be equated to the $dc_l$ representative of quantitative measurements from traditional instrumentation systems. This approach is taken for the following four reasons. First, results from PSP measurements are presented in terms of $C_P$. Second, because $C_P$ is a local surface parameter, its uncertainty varies at each point on a surface. Third, acceptable uncertainties of traditional measurement systems are usually quoted in terms of $c_l$. Fourth, because $c_l$ is a global parameter, its uncertainty is a single value. Thus, in line with convention and convenience, the uncertainty in PSP $C_P$ results will be related to an equivalent uncertainty in $c_l$. This transformation will be used in conjunction with $dC_P$ vs $M_\infty$ results from Mendoza's uncertainty analysis[3] to determine the limiting $M_\infty$ for quantitative PSP measurements.

## Analysis

The coefficient of lift may be obtained from surface pressure data as

$$c_l = \frac{1}{c}\int_0^c \Delta C_P \, dx \qquad (2)$$

where $c$ is the chord length and $\Delta C_P$ is the difference between upper and lower surface coefficients of pressure. Using the first-order process for uncertainty propagation[4] as just described, the uncertainty in $c_l$ due to the uncertainty in $C_P$ may be obtained as

$$dc_l = \frac{\partial c_l}{\partial \Delta C_P} d\Delta C_P \qquad (3)$$

The evaluation of the terms of Eq. (3) is as follows:

$$\frac{\partial c_l}{\partial \Delta C_P} = \frac{1}{c}\frac{\partial\left(\int_0^c \Delta C_P \, dx\right)}{\partial \Delta C_P} = \frac{1}{c}\int_0^c \frac{\partial \Delta C_P \, dx}{\partial \Delta C_P} = 1 \qquad (4)$$

Received Oct. 28, 1996; revision received March 21, 1997; accepted for publication March 24, 1997. 

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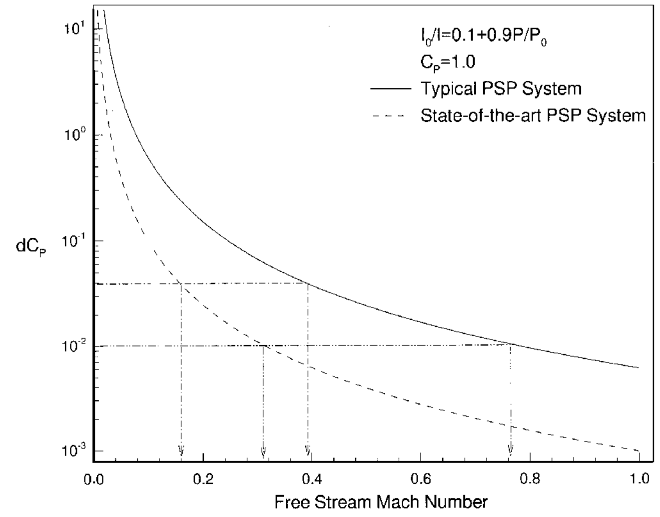


**Fig. 1 Demonstration of the technique used to determine the minimum freestream Mach number for acceptable PSP measurements based on acceptable $C_P$ uncertainties ($dC_P = 0.001$ and 0.04) for quantitative force measurements and aerodynamic assessments, respectively ($dC_P$ vs Mach number results from Ref. 3).**

$$(d\Delta C_P)^2 = \frac{1}{c}\int (dC_P)^2_{\text{lower}}\,dx + \frac{1}{c}\int (dC_P)^2_{\text{upper}}\,dx \qquad (5)$$

The difficulty with Eq. (5) is that $dC_P$ is a function of the pressure distribution over the surface and, therefore, a function of $x$. However, the worst-case PSP $dC_P$ as a function of $M_\infty$ corresponds to a $C_P$ of 1.0 (Ref. 3). Therefore, using this value of $dC_P$, Eq. (5) may be integrated and written as

$$d\Delta C_P \le \sqrt{2 dC_P^2} \qquad (6)$$

Thus, Eq. (3) may be expressed as

$$dc_l \le \sqrt{2}\, dC_P \qquad (7)$$

and $dC_P$ can be expressed in terms of $dc_l$ by taking the equality of Eq. (7) as

$$dC_P = dc_l / \sqrt{2} \qquad (8)$$

The appropriate values for $dc_l$ will be obtained by considering quantitative measurements from force balances and pressure tap instrumentation systems. Force measurements are acquired by mechanical balance mechanisms. The permissible uncertainty for these devices in terms of force coefficients is 0.25% for low-speed flow conditions.[6] This results in a $dc_l$ of $\pm 0.002$. Aerodynamic assessments of wing sections include determining points of stagnation, separation, maximum and minimum pressure, shock wave, and vortex burst. Quantitative determination of these characteristic points based on pressure tap data correspond to a $dc_l$ of $\pm 0.05$ (Ref. 7). Thus, using Eq. (8), the acceptable $dC_P$ for PSP measurements with respect to quantitative force measurements and aerodynamic assessments of wing sections is

$$dC_P \le 0.001 \qquad \text{and} \qquad dC_P \le 0.04$$

respectively.

Using these $dC_P$ values with results of $dC_P$ vs $M_\infty$ the limiting Mach number for quantitative PSP measurements can be determined. This technique is demonstrated in Fig. 1, which shows $dC_P$ vs $M_\infty$ results[3] for typical and state-of-the-art PSP instrumentation systems. Figure 1 shows the minimum $M_\infty$ for acceptable measurements corresponding to a $dC_P$ of 0.001 is 0.75 and 0.30 for typical and state-of-the-art PSP systems, respectively. Figure 1 also shows the minimum $M_\infty$ for acceptable measurements corresponding to a $dC_P$ of 0.04 is 0.40 and 0.15 for typical and state-of-the-art PSP systems, respectively.

## Conclusions

The minimum $M_\infty$ for quantitative PSP measurements of pressure can be determined from the desired uncertainty in $c_l$ and $C_P$. The limiting $M_\infty$ for quantitative results of pressure derived force coefficients is 0.75 and 0.30 for typical and state-of-the-art PSP instrumentation systems, respectively. The limiting $M_\infty$ for quantitative results of pressure derived aerodynamic assessments is 0.40 and 0.15 for typical and state-of-the-art PSP instrumentation systems, respectively.

## Acknowledgments

This work was supported in part through the NASA Graduate Student Researchers Program and the National Research Council. D. R. Mendoza thanks his research advisors, Maurice Holt and Gary Chapman of the University of California and B. G. McLachlan of the NASA Ames Research Center, for their guidance, advice, and friendship during this work.

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# Resolution Effects in Chaotic Velocity Field Reconstruction from Passive Scalar Data

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## Introduction

RECENTLY, proposals have been made for determining complete velocity fields from the measurement of one or more passive scalars.[1–3] It is unclear how chaotic and/or noisy fluctuations in time-dependent experimental scalar data will affect the accuracy of these methods when the scalar data are not sufficiently resolved. This is of particular importance if one intends to use them to determine turbulent velocity fields as in Refs. 2 and 3.

Numerous concerns can be raised regarding shortcomings of approaches such as those proposed in Refs. 1–3; for the sake of brevity we will simply note here that many of these have been addressed for specific cases, such as low-Reynolds-number ($Re$) steady flow,[4] and qualitatively for more general, i.e., time-dependent, turbulent, situations.[3] But quantitative evaluations in a well-controlled experiment seem not to be available. The present Note is intended to provide a first step in filling this void.

Received Aug. 21, 1996; revision received Feb. 25, 1997; accepted for publication April 4, 1997. 

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